# Logarithmic Functions

Logarithms are inverses of exponential functions (just like multiplication is the inverse of division). They undo each other.

A **logarithm** base  of a positive number satisfies the following definition.

For

is equivalent to

where,

we read  as, “the logarithm with base  of” or the “log base  of ."

The logarithm  is the exponent to which  must be raised to get .

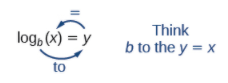
Also, since the logarithmic and exponential functions are inverses and switch the  and  values, the domain and range of the exponential function are interchanged for the logarithmic function. Therefore,

the domain of the logarithm function with base  is .

the range of the logarithm function with base  is .

# Converting Between Logarithmic and Exponential Forms

Using what we know about exponential and logarithmic functions, we can convert between the two forms. To convert, identify the base , the exponent , and the output .



Examples

1. Write the following logarithmic equations in exponential form.
2. Write the following exponential equations in logarithmic form.

# Evaluating Logarithms

Examples:

1. Evaluate the following logarithms without using a calculator.
2. Evaluate the following expressions without using a calculator.

# Using Common and Natural Logarithms

Sometimes we may see a logarithm written without a base. In this case, we assume that the base is 10. In other words, the expression  means .

A **common logarithm** is a logarithm with base 10.  We write  simply as  The common logarithm of a positive number  satisfies the following definition.

For ,

 is equivalent to

We read  as, “the logarithm with base 10 of  ” or “log base 10 of .”

The logarithm  is the exponent to which 10 must be raised to get .

The most frequently used base for logarithms is ; they are called **natural logarithms**. The base   logarithm, , has its own notation .

A **natural logarithm** is a logarithm with base . We write  simply as . The natural logarithm of a positive number  satisfies the following definition.

For ,

 is equivalent to

We read as, “the logarithm with base  of ” or “the natural logarithm of .”

The logarithm   is the exponent to which  must be raised to get .

Since the functions  and  are inverse functions,   for all  and   for .

Examples:

1. For each of the following, use the definition of common and natural logarithms to simplify.
2. For each of the following, evaluate the expression without using a calculator.
3. The amount of energy released from one earthquake was 500 times greater than the amount of energy released from another. The equation  represents this situation, where   is the difference in magnitudes on the Richter Scale. To the nearest thousandth, what was the difference in magnitudes?